Math 1011 Honors Ch2 Quadratics: Challenging 2 Problems Involving Discriminants

Example 3. (AMC Problem 24) If a and b are positive real numbers and each of the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ has real roots, then the smallest possible value of $a + b$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Example 4. Consider equations of the form $3x^2 + (b + 1)x + c = 0$. How many such equations have distinct real roots and have coefficients b selected from the set of odd integers $\{3, 5, 7, 9, 11, 13, 15\}$ and c selected from any positive integer?

(A) 64 (B) 66 (C) 108 (D) 107 (E) 32

Example 5. Find the real values of a if at least one of the following three equations has the real roots.

$x^2 + 4ax - 4a + 3 = 0$

$x^2 + (a - 1)x + a^2 = 0$

$x^2 + 2ax - 2a = 0$

(A) $a \le -\frac{3}{2}$ or $a \ge -1$ (B) $-2 < a < 0$ (C) $-\frac{3}{2} < a \le \frac{1}{2}$ (D) $a > \frac{1}{3}$ or $a < -1$

(E) $-\frac{3}{2} < a \le \frac{1}{2}$

Example 6. Find the real values of m such that the equation $4x^2 + (m - 2)x + m - 5 = 0$ has the negative real roots.

Example 7. Find the real values of m such that the equation $(k^2 - 4)x^2 - 4(k + 2)x + 4 = 0$ has two real roots.

(A) $k > -2$ (B) $k > 2$ (C) $k \ne 2$ and $k > -2$ (D) $k < -2$ and $k \ne -2$

(E) $k = \pm 2$

Example 8. Find the real values of a and b such that the equation

$$x^2 - (a+b)x + \frac{a^2 + 2b^2 - 2b + 1}{2} = 0$$ has two real roots.

Example 10. (AMC) If x is real and $4y^2+4xy+x+6=0$, then the complete set of values of x for which y is real, is:

(A) $x \le -2$ or $x \ge 3$ (B) $x \le 2$ or $x \ge 3$

(C) $x \le -3$ or $x \ge 2$ (D) $-3 \le x \le 2$ (E) $-2 \le x \le 3$

Example 11. Solve the equation for real values of x and y:

$x^2-4xy+5y^2+2x-8y+5=0$.

Example 12. Solve for real values of x and y: $10x^2+6xy+y^2-14x-4y+5=0$.

Example 16. (USAMO) The sum of 5 real numbers is 8 and the sum of their squares is 16. What is the largest possible value for one of the numbers?

Example 17. Find the range of k such that the system of equations has real solutions.

$$\begin{cases} x^2-xy+y^2=k & (1) \\ x^2+y^2=5 & (2) \end{cases}$$